Mapping Flows in Bipartite Networks

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The Problem: Community Detection in Bipartite Networks

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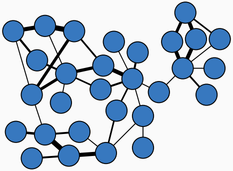
Treat as unipartite



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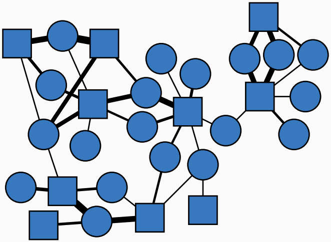

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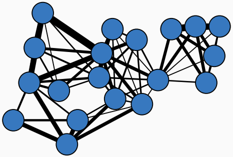
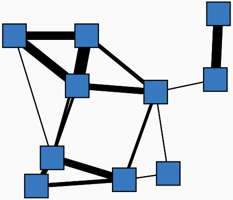


Unipartite projections

⇒



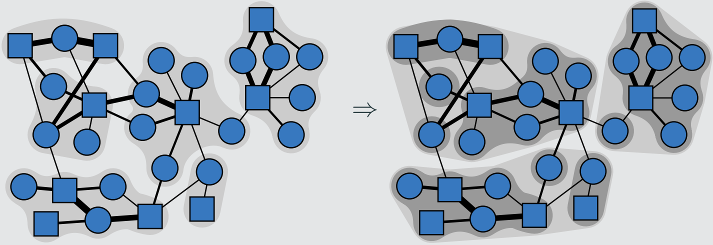
The Problem: Community Detection in Bipartite Networks

Our Solution

Teach the map equation to recognise bipartite networks.

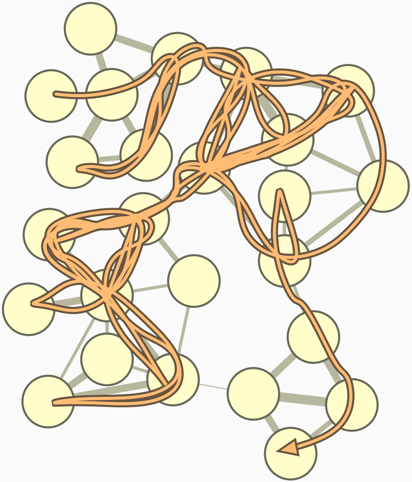
The Benefits

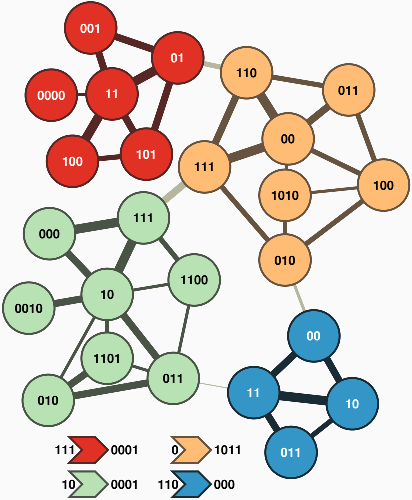
- We use all available data efficiently
- This improves the compression and we find more regularities
- We increase the resolution and explore different scales



Quick Overview:
The Map Equation Framework

The Map Equation Framework





$$L(M) = qH(\mathcal{Q}) + \sum_{m \in M} p_m H(\mathcal{P}_m)$$

Our Solution:
The Bipartite Map Equation

The Bipartite Map Equation

Idea

Reflect the bipartite network structure in the coding scheme.

Key insight

Random walks must alternate between node types.

$\rightarrow$ network flow is divided evenly between node types,

$\rightarrow$ one node type is visited in even steps, the other in odd steps,

$\rightarrow$ there are two random processes:

- X: the current node
- Y: the current node type
- then we can use Bayes' rule: $H(X|Y) = H(X) - H(Y) + H(Y|X)$

$$L(\mathsf{M}_1) = \underbrace{H(\mathcal{P})}_{H(X)} = \underbrace{1}_{H(Y)} + \underbrace{\frac{1}{2}H\left(\mathcal{P}^L\right) + \frac{1}{2}H\left(\mathcal{P}^R\right)}_{H(X|Y)}$$

$\rightarrow$ plug this into the map equation...

The Bipartite Map Equation

$$L(\mathsf{M}) = qH(\mathcal{Q}) + \sum_{\mathsf{m}\in\mathsf{M}} p_{\mathsf{m}} H(\mathcal{P}_{\mathsf{m}})$$

$$\downarrow$$

$$\begin{aligned} L_B(\mathsf{M}) = {} & q^L H\left(\mathcal{Q}^L\right) + \sum_{\mathsf{m}\in\mathsf{M}} p_{\mathsf{m}}^L H\left(\mathcal{P}_{\mathsf{m}}^L\right) \\ & + q^R H\left(\mathcal{Q}^R\right) + \sum_{\mathsf{m}\in\mathsf{M}} p_{\mathsf{m}}^R H\left(\mathcal{P}_{\mathsf{m}}^R\right) \end{aligned}$$

Problem

In sparse networks, knowing the node type comes close to knowing the exact node $\rightarrow$ encoding close to the entropy rate of the Markov process without identifying modular structure.

A key ingredient of the map equation is forgetting the right amount!

Our Solution 2.0:
The Bipartite Map Equation
with Varying Node-Type Memory

The Bipartite Map Equation with Varying Node-Type Memory

Idea

Reflect the bipartite network structure in the coding scheme **and forget node types at rate** α.

Then

- Random walks still alternate between node types
- Random walker "confuses" node types with probability α
  $\rightarrow$ new visit rates: $p_u \rightsquigarrow p_u^\alpha = ((1-\alpha)\,p_u, \alpha p_u)$ for left nodes u
  $p_v \rightsquigarrow p_v^\alpha = (\alpha p_v, (1-\alpha)\,p_v)$ for right nodes v
- Network flow is still evenly divided between node types!
- Before: $H(Y|X) = 0$
  Now: $H(Y|X) = H(\{\alpha, 1-\alpha\}) = H_\alpha$

$$L(\mathsf{M}_1) = \underbrace{H(\mathcal{P}_1)}_{H(X)} = \underbrace{1}_{H(Y)} - \underbrace{H_\alpha}_{H(Y|X)} + \underbrace{H(\mathcal{P}_1^\alpha)}_{H(X|Y)}$$

$\rightarrow$ plug this into the map equation...

The Bipartite Map Equation with Varying Node-Type Memory

$$L\left(\mathsf{M}\right) = qH\left(\mathcal{Q}\right) + \sum_{\mathsf{m}\in\mathsf{M}} p_{\mathsf{m}}H\left(\mathcal{P}_{\mathsf{m}}\right)$$

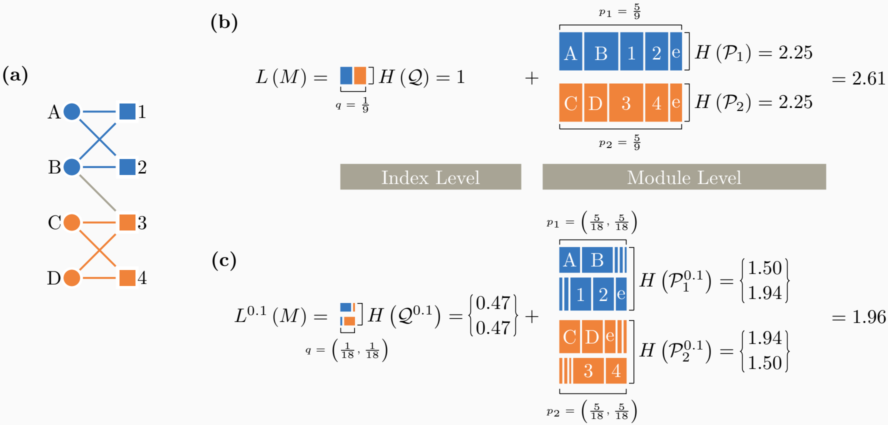
$$\downarrow$$

The Bipartite Map Equation with Varying Node-Type Memory

$$L^{\alpha}\left(\mathsf{M}\right) = q^{\alpha}H\left(\mathcal{Q}^{\alpha}\right) + \sum_{\mathsf{m}\in\mathsf{M}} p^{\alpha}_{\mathsf{m}}H\left(\mathcal{P}^{\alpha}_{\mathsf{m}}\right)$$

For $\alpha = \frac{1}{2}$, we recover the standard map equation!

The Bipartite Map Equation with Varying Node-Type Memory



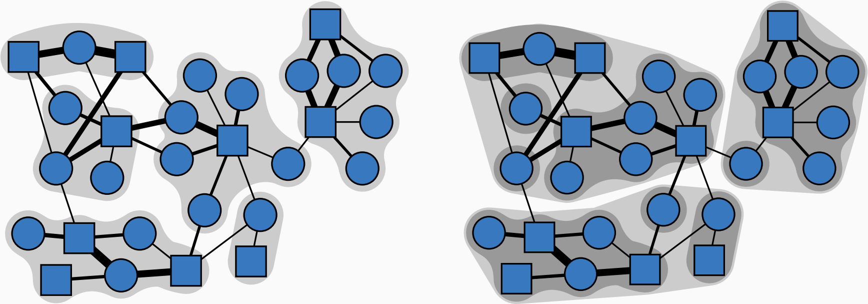
Results

Results

The Bipartite Map Equation with Varying Node-Type Memory

$$L^{\alpha}(\mathsf{M}) = q^{\alpha} H(\mathcal{Q}^{\alpha}) + \sum_{\mathsf{m} \in \mathsf{M}} p_{\mathsf{m}}^{\alpha} H(\mathcal{P}_{\mathsf{m}}^{\alpha})$$

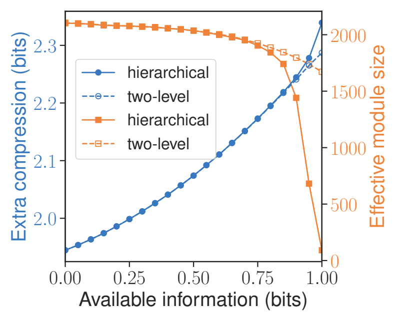
$\alpha = \frac{1}{2}$ $\qquad$ $\alpha = \frac{1}{6}$



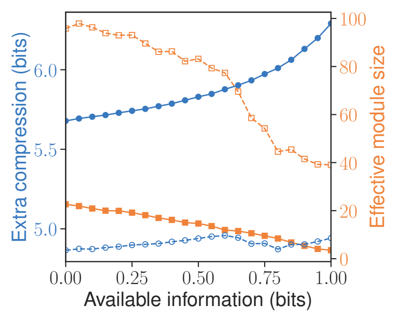
$\Rightarrow$ we can change the resolution and explore different scales!

Results

(a) Last.fm user-song network.



(b) IMDb actor-movie network.



Extra compression: compared to one-module partition

Available information (bits) $= 1 - H_\alpha$

Effective module size $= 2^{H(S)}$

Conclusion

Results

Problem

Most community detection methods address unipartite networks.

Idea

Extend the map equation framework to reflect the regularities in bipartite networks.

Benefits

We use node-type information efficiently and increase the compression. Further, by adjusting the resolution, we explore community structures at different scales.

Acknowledgements

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