Mapping flows in bipartite networks

The bipartite map equation
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The map equation

- Information-theoretic objective function
- Exploits duality between compression and finding patterns
- Models information flows with a random walk

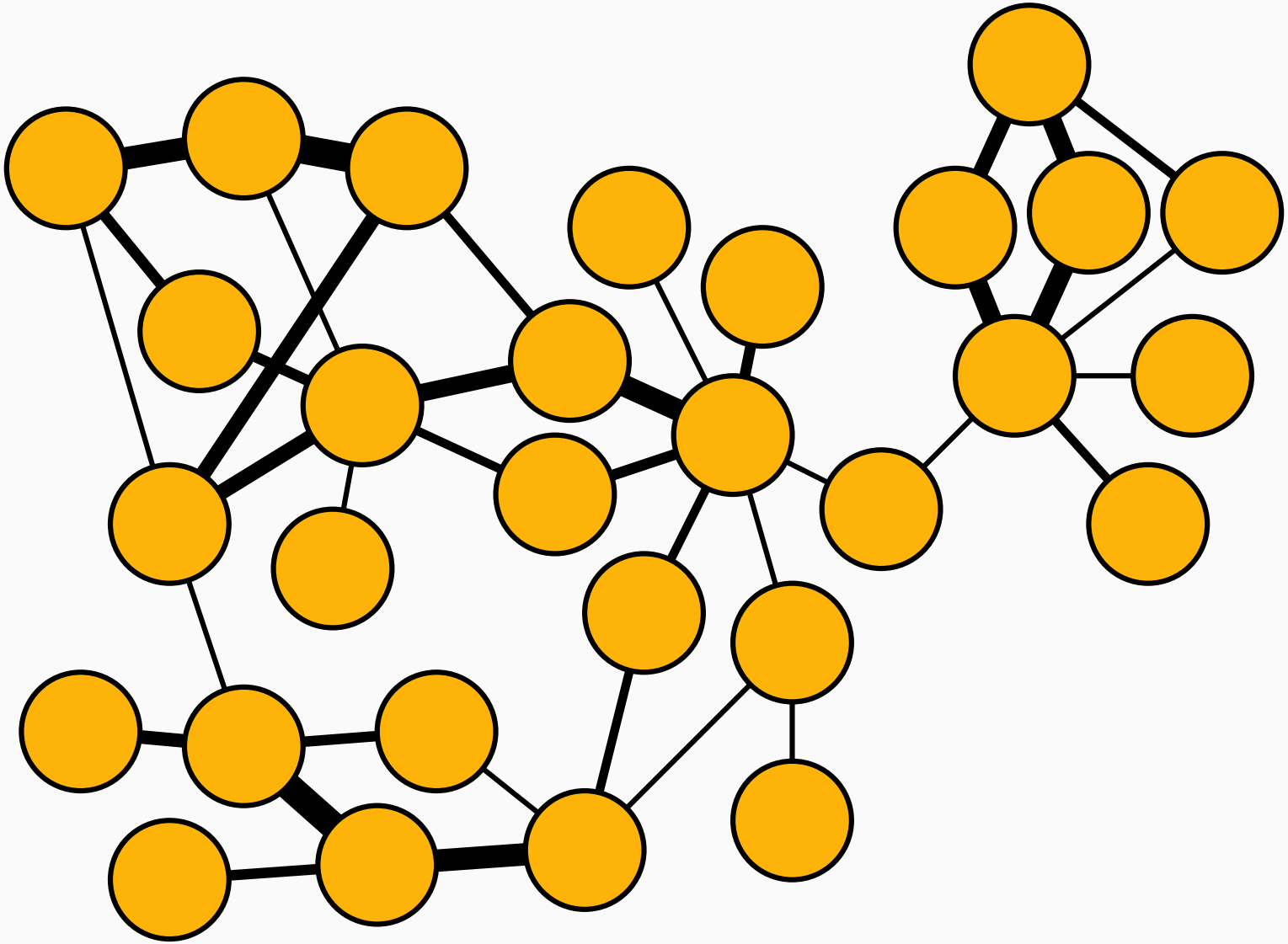
Communication game

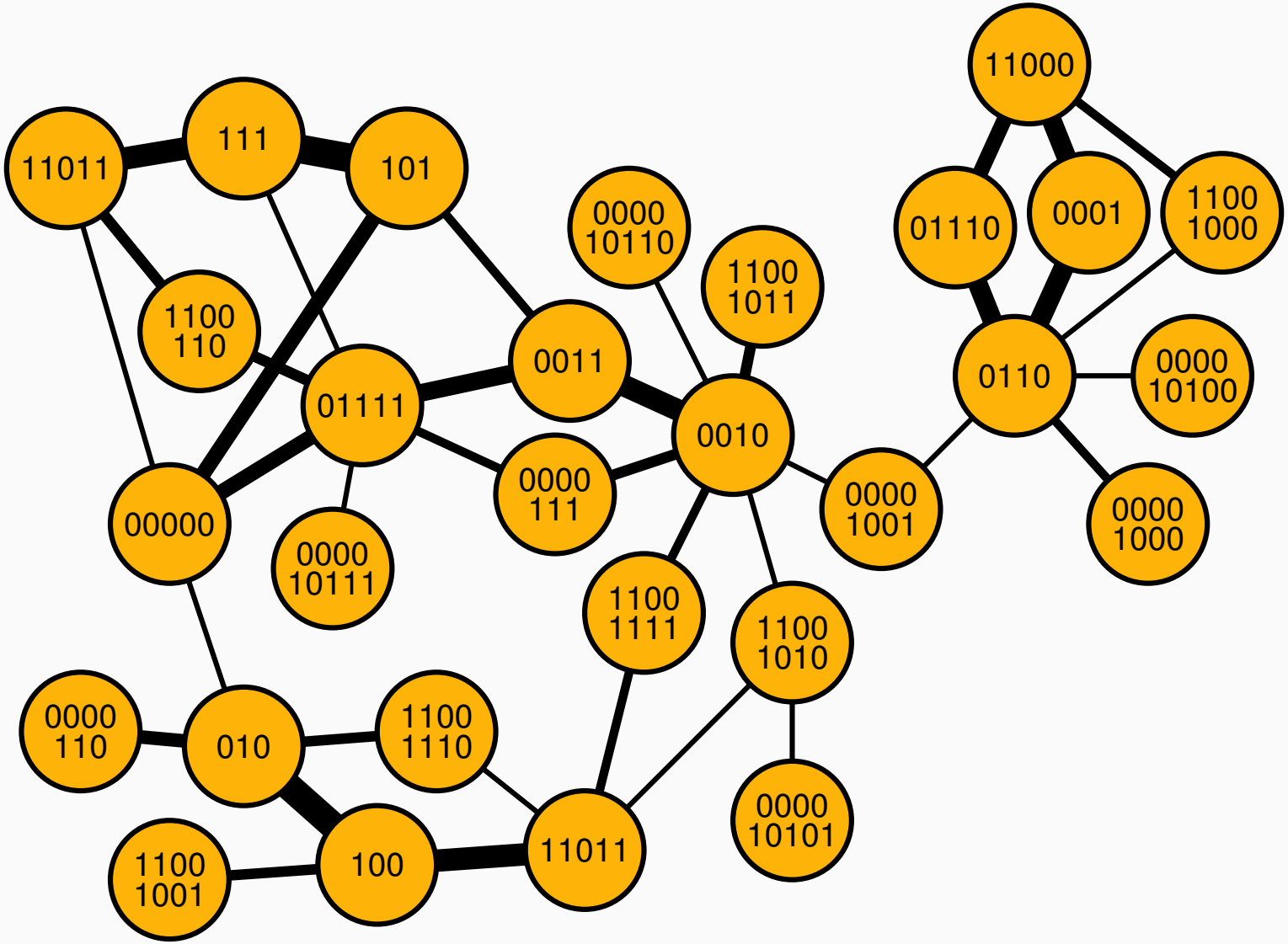
- Sender
- Receiver
- Random walker in a network
- Sender updates receiver about random walker location

Question

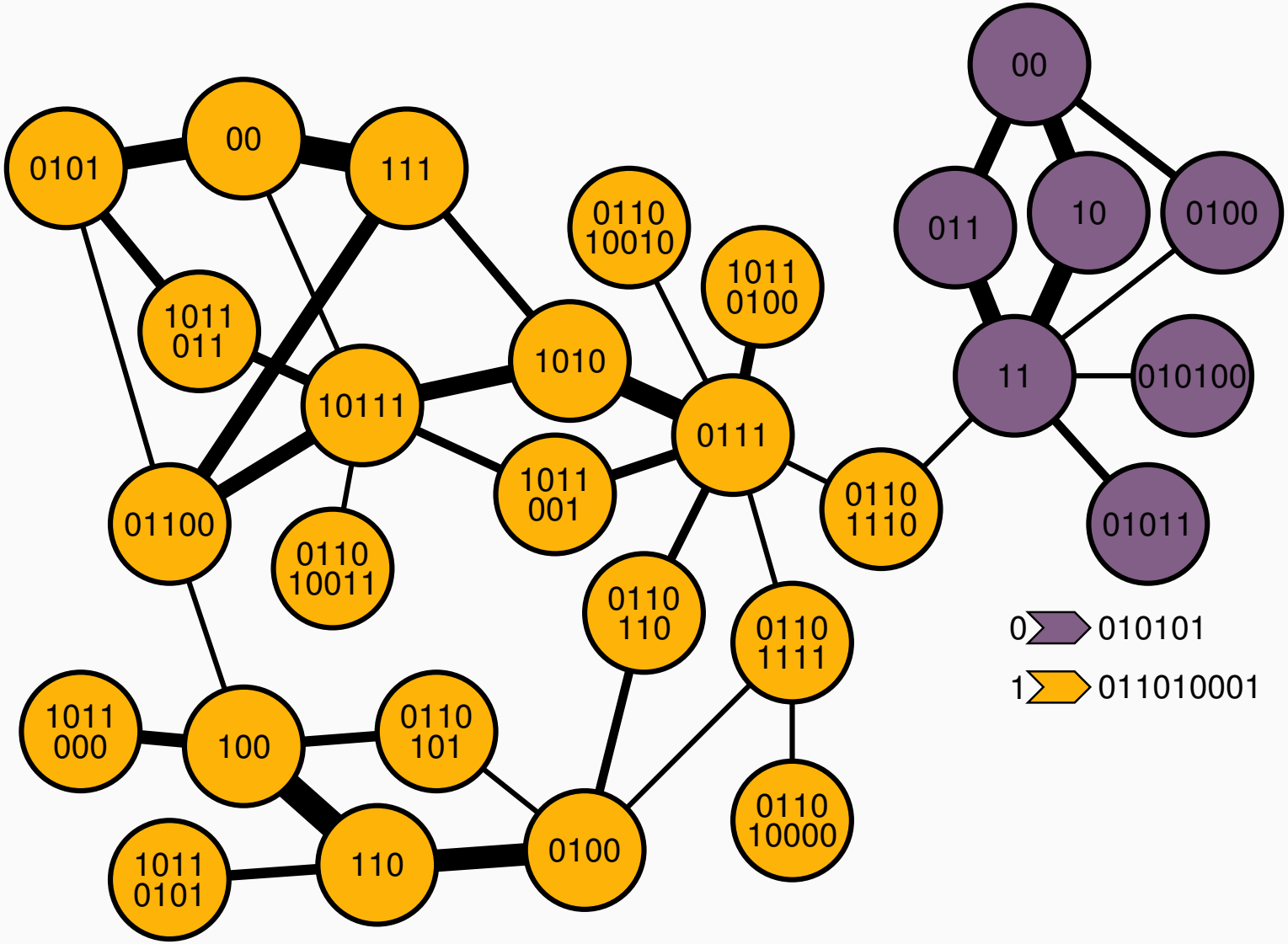
How can we design an efficient coding scheme for this?

efficient = minimise $\underbrace{\text{average per-step description length}}_{\text{“code length”}}$

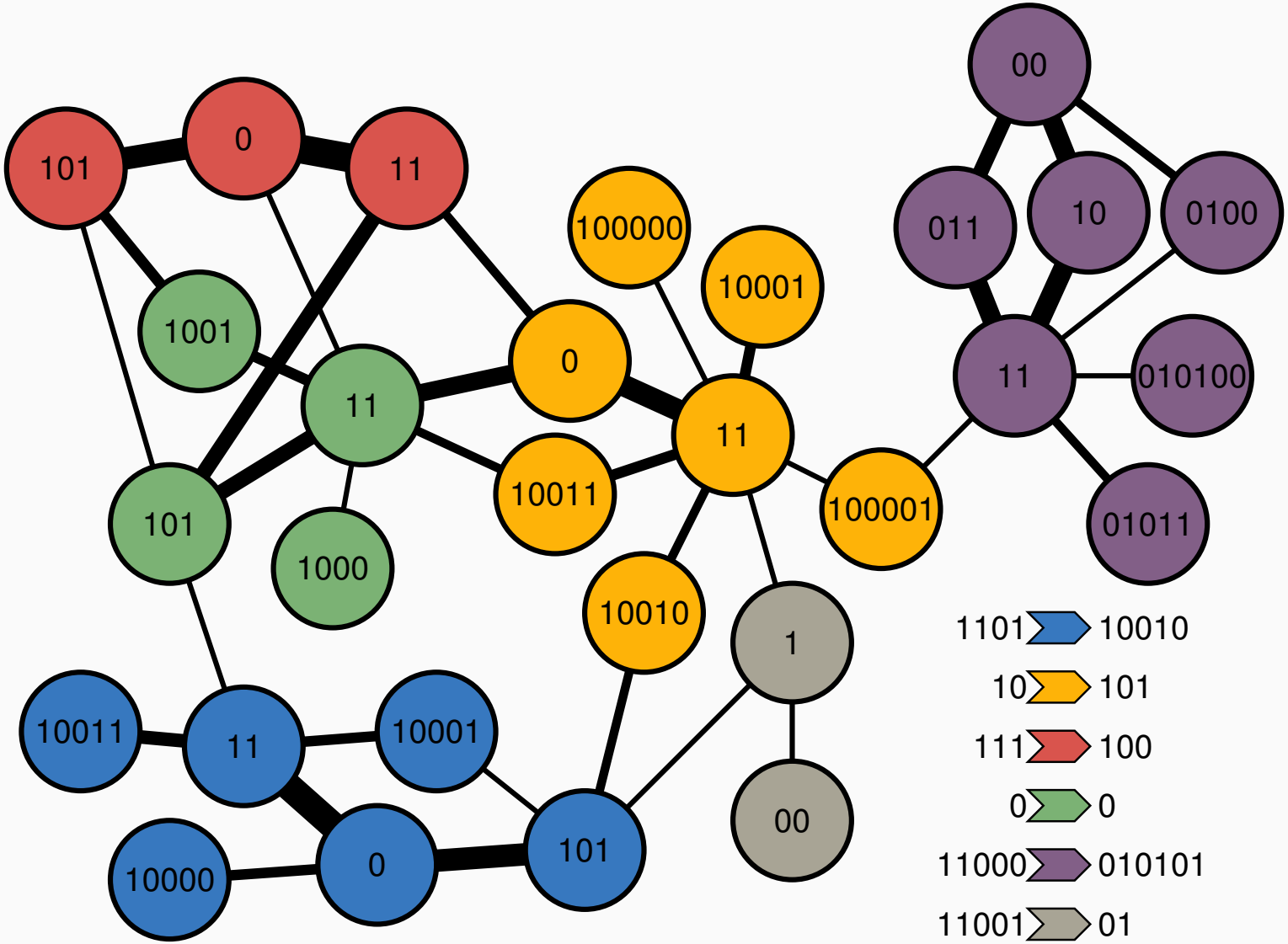




What is the code length? $L(\mathsf{M}) = H(\mathcal{P}) = 3.85\text{bits}$



$$L(\mathsf{M}) = qH(\mathcal{Q}) + p_1 H(\mathcal{P}_1) + p_2 H(\mathcal{P}_2) = 3.22\text{bits}$$



$$L(\mathsf{M}) = qH(\mathcal{Q}) + \sum_{i=1}^{m} p_i H(\mathcal{P}_i) = 2.24\text{bits}$$

The map equation

$$L(\mathsf{M}) = qH(\mathcal{Q}) + \sum_{i=1}^{m} p_i H(\mathcal{P}_i)$$

...can be generalised through recursion

What is the problem?

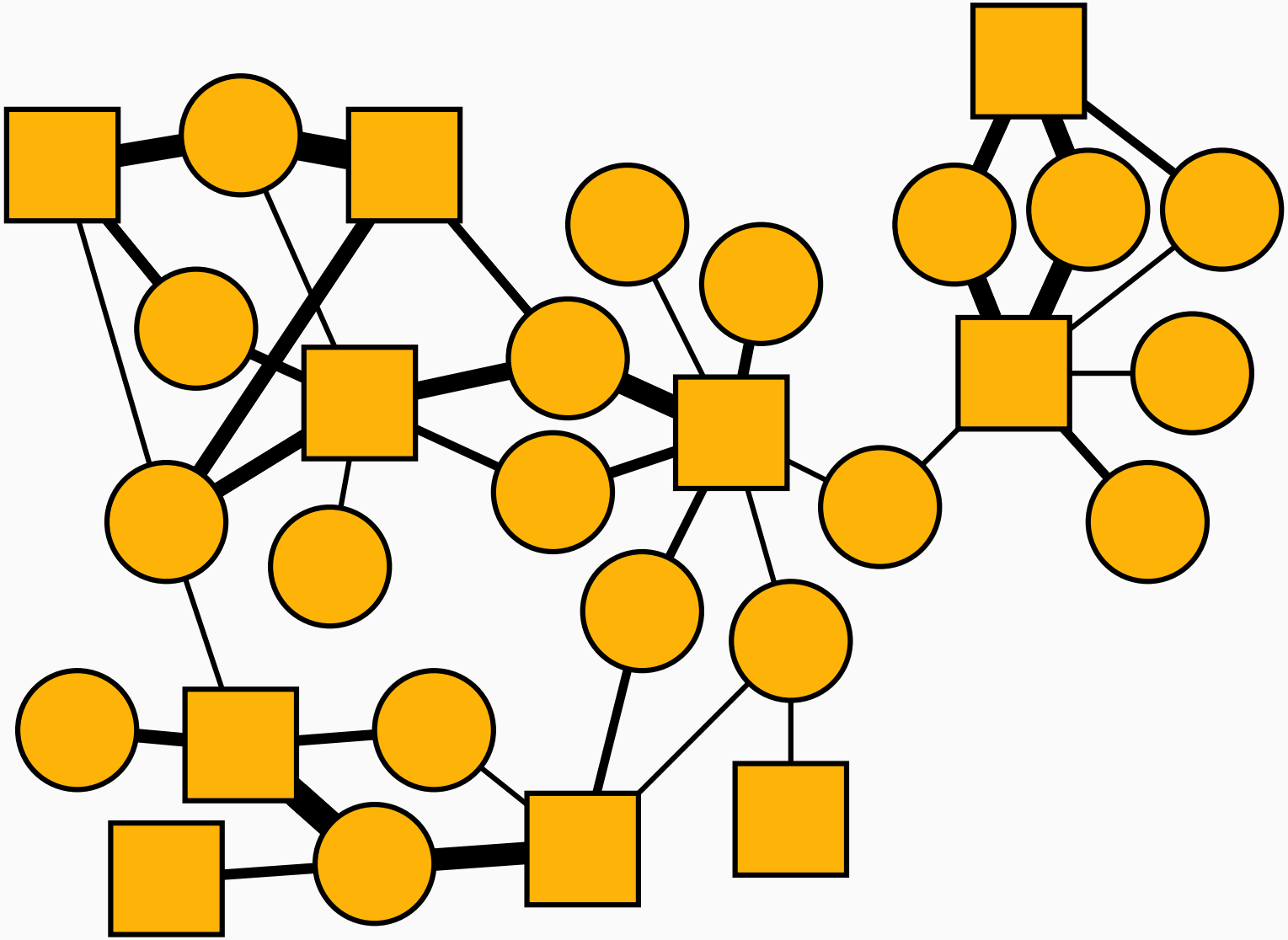
- Coding scheme for steps between any pair of nodes
- Random walks in **bipartite** networks **alternate** between node types, this excludes many transitions

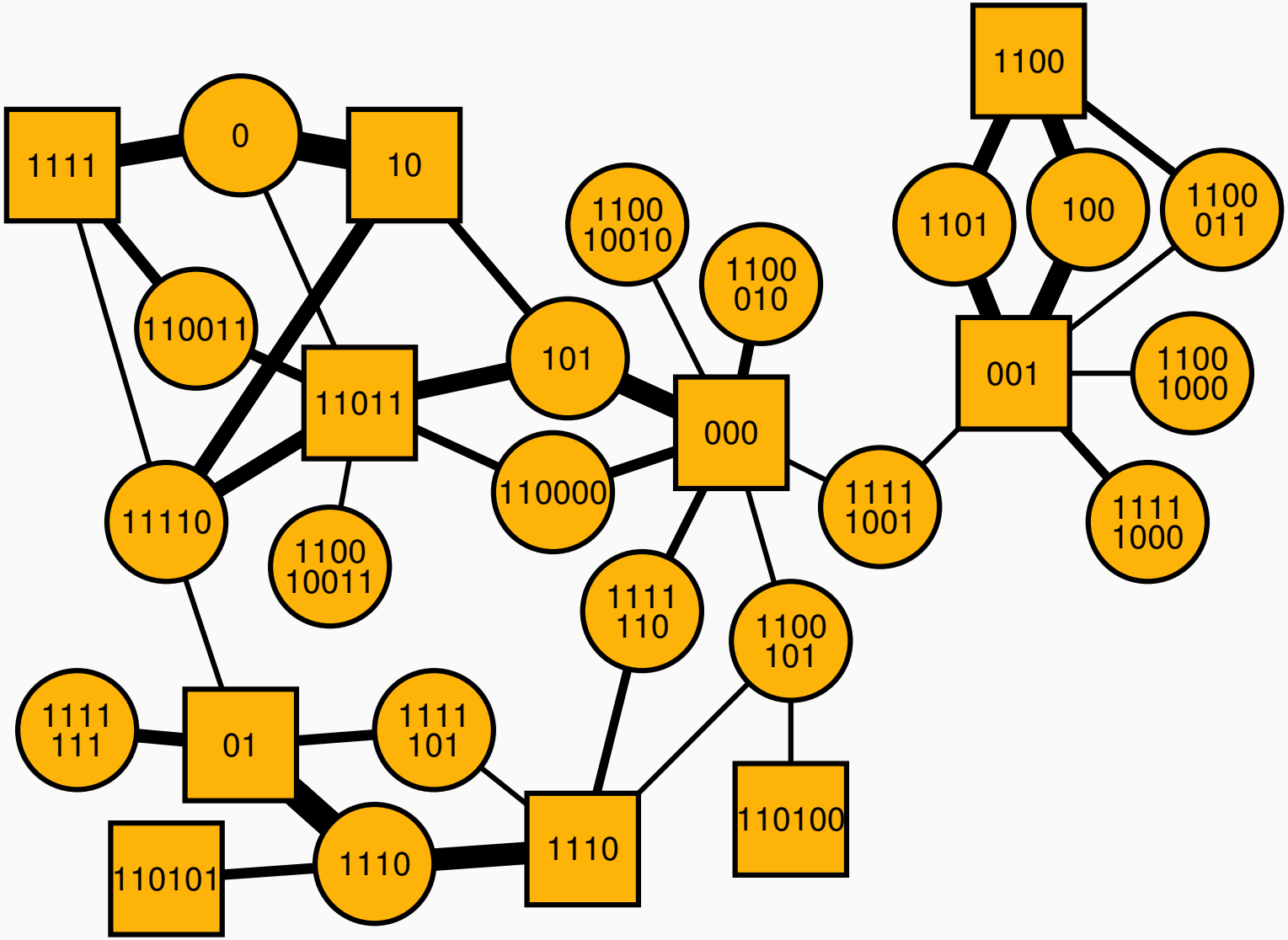
→ The map equation produces longer codes than necessary

Does that mean we cannot use it on bipartite networks?

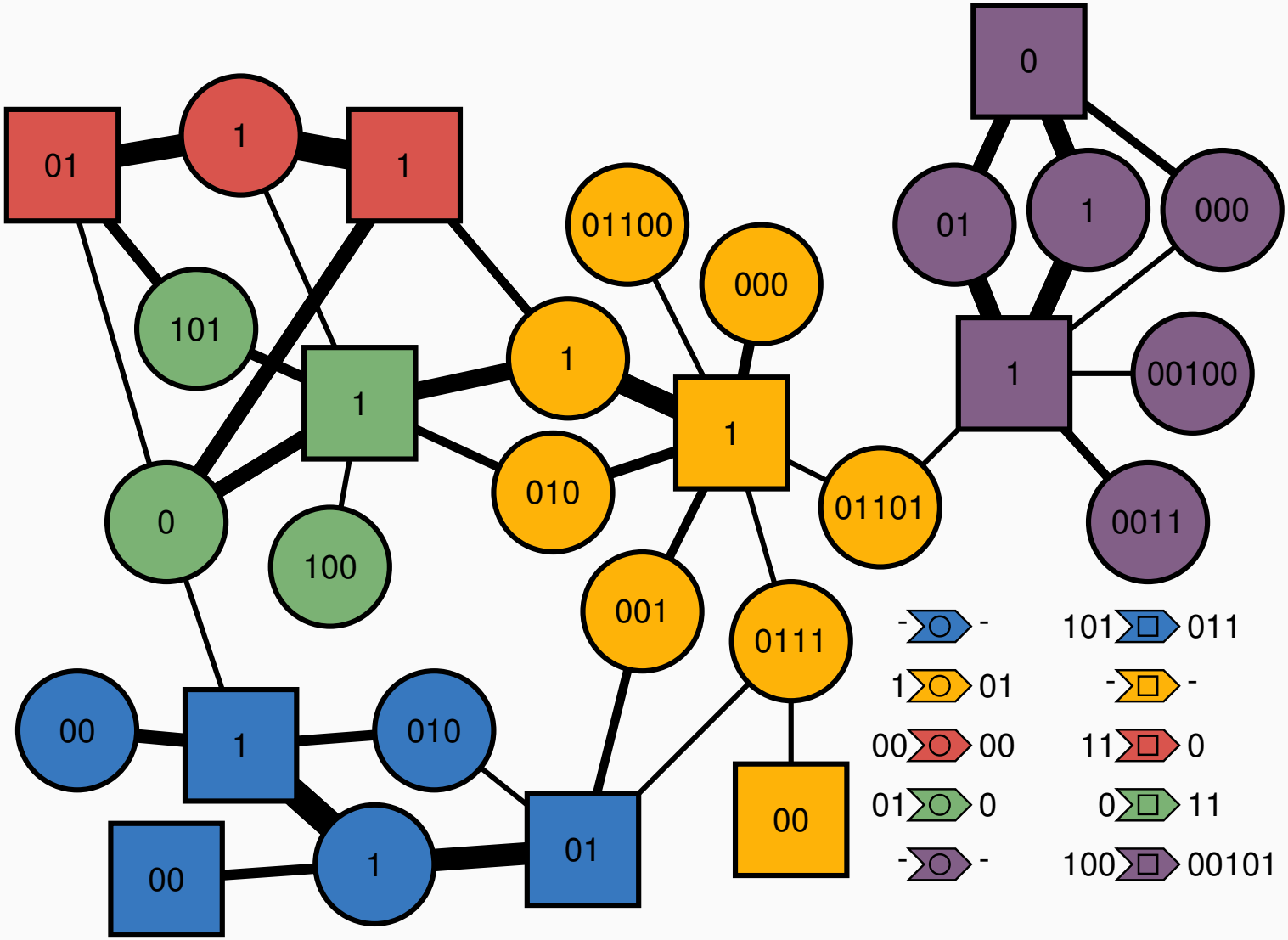
- We can make a unipartite projection, but that's lossy...
- We can use the map equation directly, but that's wasteful...

→ How can we do better?





$$L(\mathsf{M}) = \frac{1}{2}H\left(\mathcal{P}^1\right) + \frac{1}{2}H\left(\mathcal{P}^2\right) = 2.85\text{bits}$$



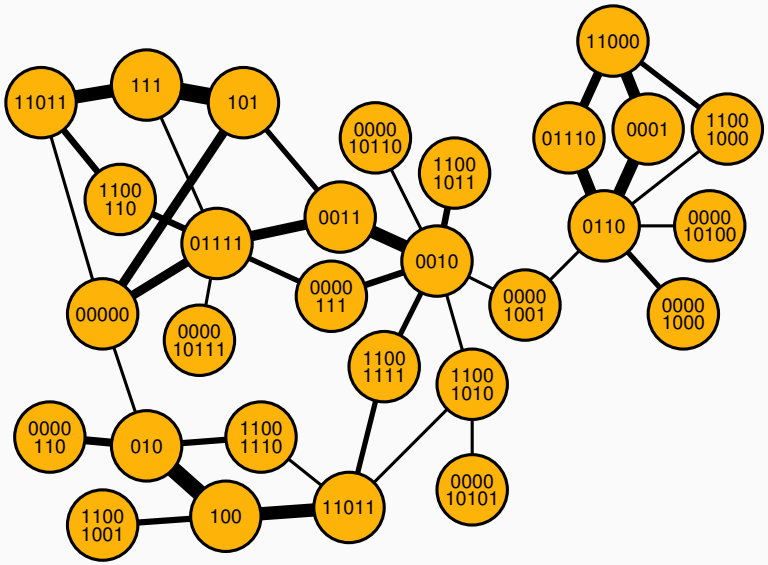
$$L_B(\mathsf{M}) = q^1 H\left(\mathcal{Q}^1\right) + \sum_{i=1}^{m} p_i^1 H\left(\mathcal{P}_i^1\right) + q^2 H\left(\mathcal{Q}^2\right) + \sum_{i=1}^{m} p_i^2 H\left(\mathcal{P}_i^2\right)$$

$$= 1.18\text{bits}$$

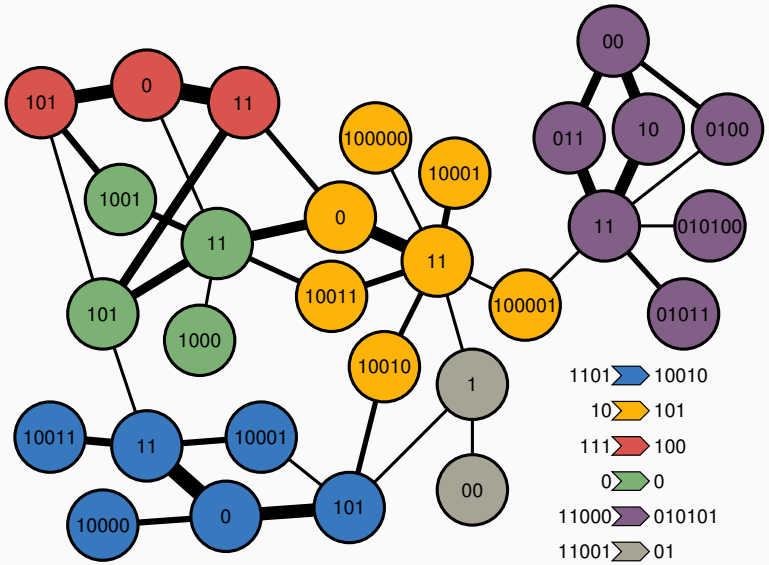
The bipartite map equation

$$L_B(\mathsf{M}) = q^1 H\left(\mathcal{Q}^1\right) + \sum_{i=1}^{m} p_i^1 H\left(\mathcal{P}_i^1\right) + q^2 H\left(\mathcal{Q}^2\right) + \sum_{i=1}^{m} p_i^2 H\left(\mathcal{P}_i^2\right)$$

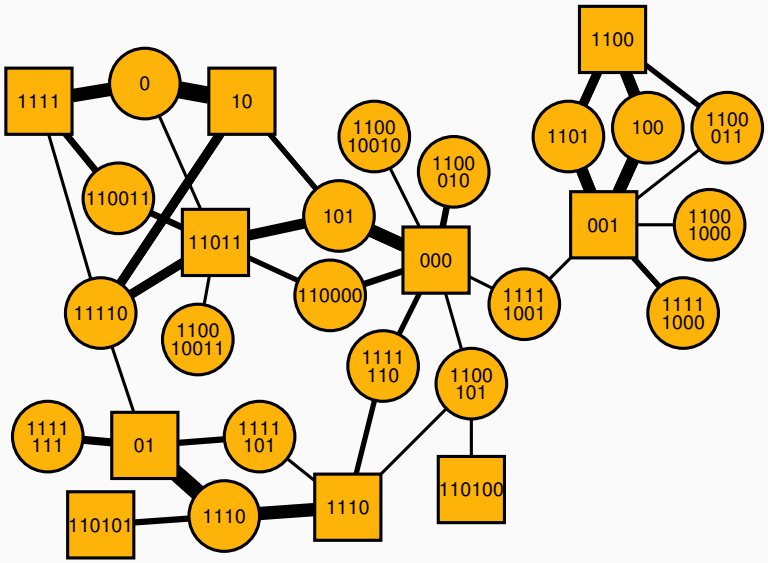
- uses pairs of code books
- alternates between code book types



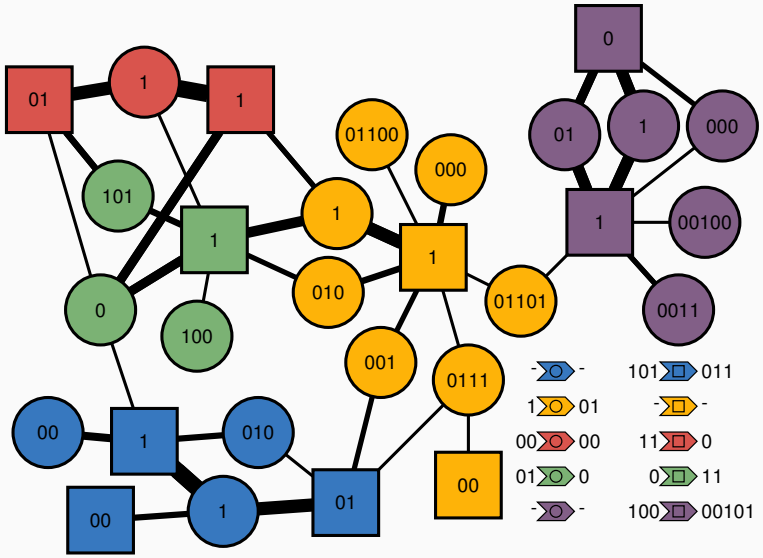
3.85 bits



2.24 bits

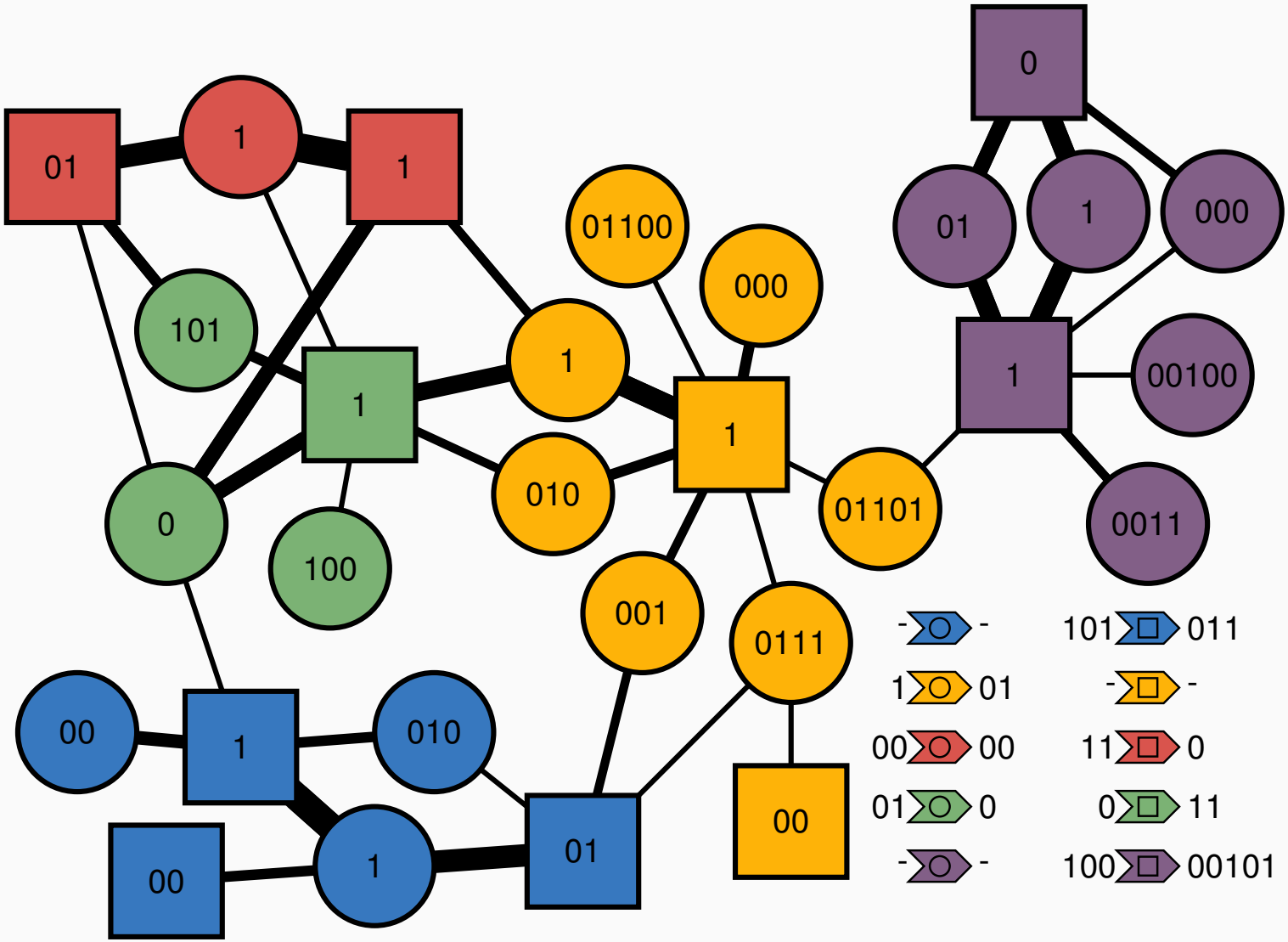


2.85 bits



1.18 bits

What is going on?



Implemented in Infomap (soon public on GitHub)

Name	n_1	n_2	l	L	L_B
IMDb actor-movie	124,414	374,511	1,460,791	11.99	10.88
YouTube user-group	88,490	25,007	286,913	10.25	9.14
GitHub Contest	39,845	99,907	417,361	11.23	10.09
Wiktionary (de)[W]	5,354	144,710	686,661	11.23	10.20
Wiktionary (en)[W]	26,719	2,091,461	5,569,967	12.14	11.08
Virus-host interact.	41	288	433	4.89	3.64
Arroyo Goye[W]	27	8	41	2.70	1.65
Fonseca Ganade[W]	19	10	38	2.24	1.18
Norwegian directors	212	854	1,148	3.83	2.39
Scottish companies	86	131	348	5.17	3.88
Kernel mailing list[W]	34,490	330,155	591,199	9.61	8.54
Last.fm song[W]	1,084,620	992	4,413,834	12.33	11.30
Stack Overflow	524,670	80,492	1,280,982	11.83	10.64
UC Irvine forum[W]	897	520	7,087	8.16	7.14
PGHF meetup	4,989	4,611	39,501	10.90	9.83
SIAM conference	10,018	19	15,533	7.94	6.78
Reuters story-word	19,757	38,677	978,446	13.44	12.40
Wikipedia excellent	2,780	273,959	2,941,902	13.63	12.51

Results

The bipartite map equation

- provides better compression,
- finds more, smaller modules in deeper hierarchies, and
- makes community structures more bipartite.

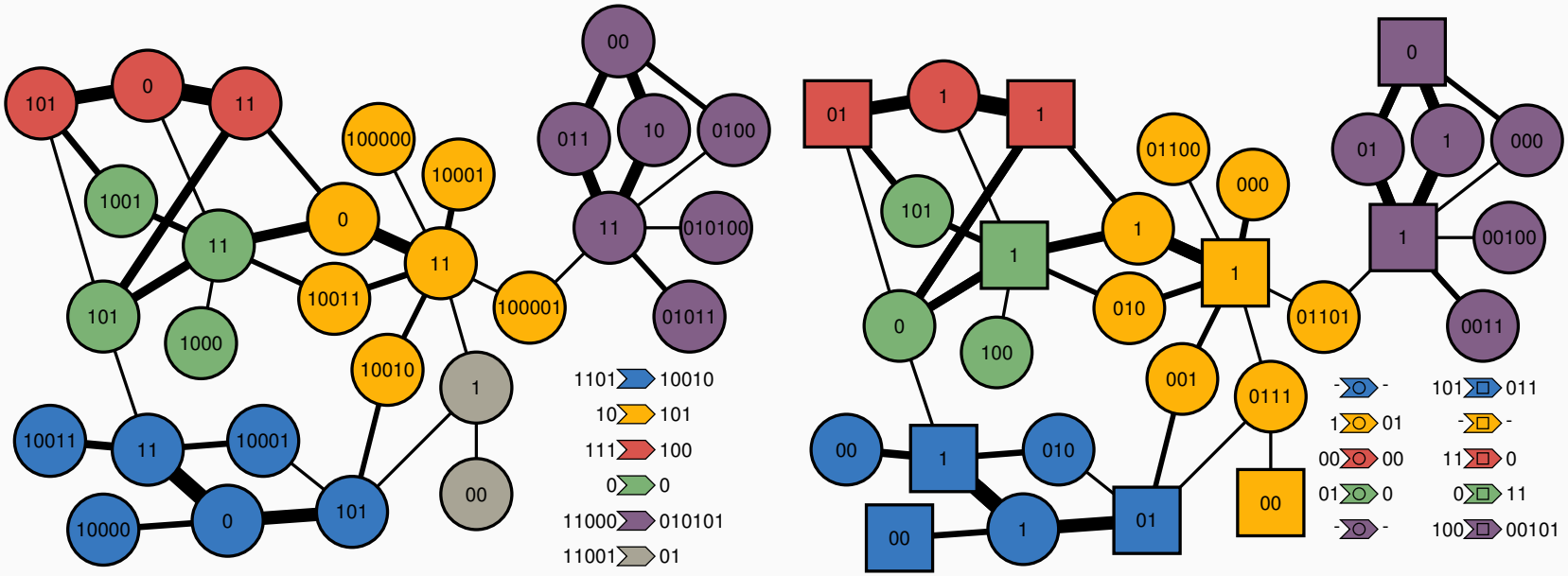
Community Balance

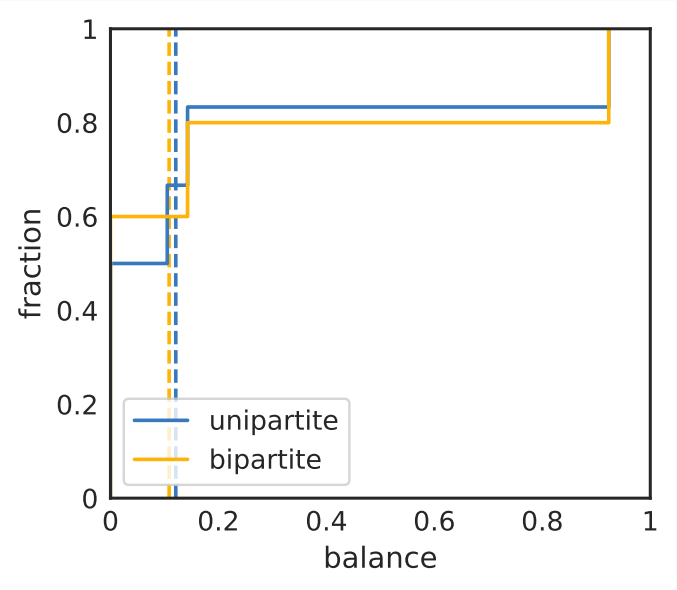
$$b_i = 1 - \left| \frac{q_i^1 - q_i^2}{q_i^1 + q_i^2} \right|$$

Partition Balance

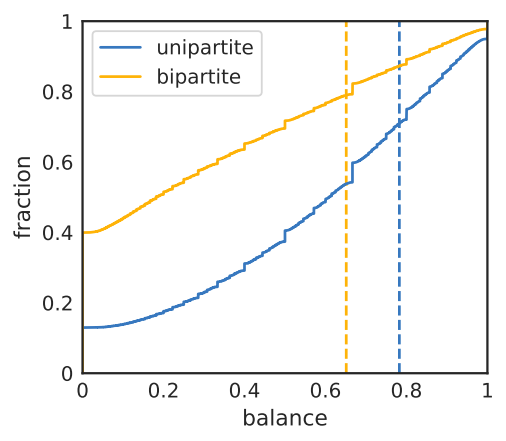
$$b = \sum_i \frac{p_i b_i}{\sum_i p_i}$$

Cumulative distribution of module balance

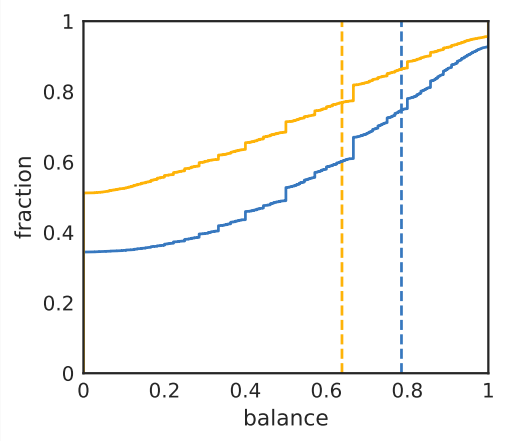




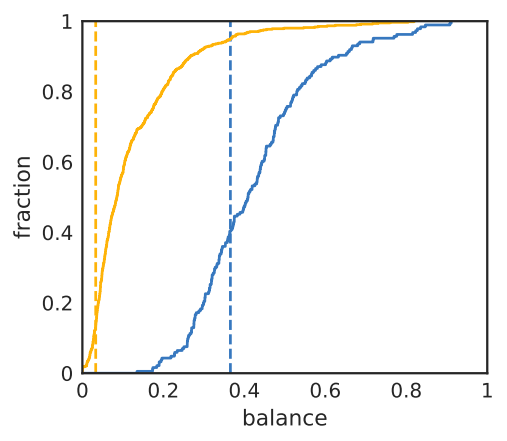
Cumulative distribution of module balance



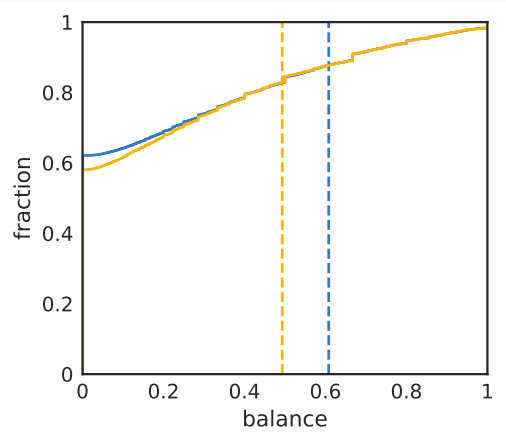
IMDb actor-movie



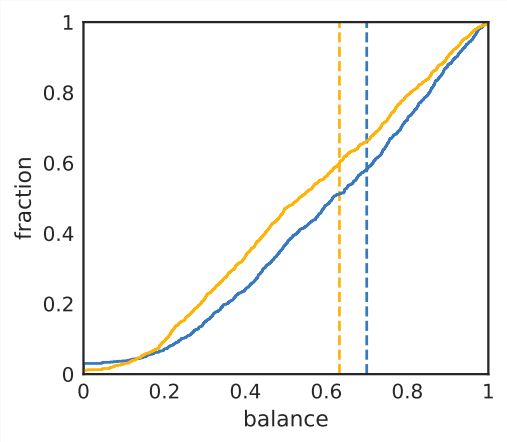
YouTube user-group



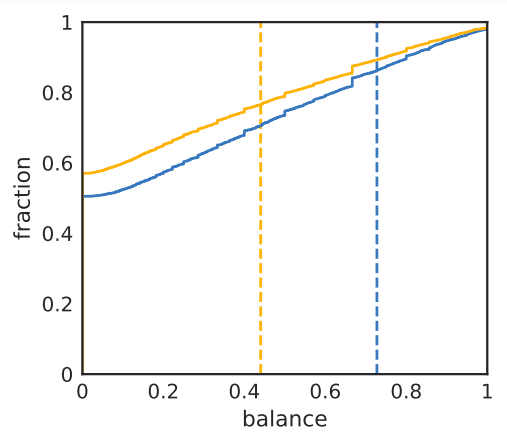
Wikipedia excellent article-word



Wiktionary (en)^W author-article



Last.fm user-song^W



Kernel mailing list^W user-thread

Summary

- Based on the alternating trajectory of random walks in bipartite networks, we use an alternating coding scheme to derive the bipartite map equation
- The bipartite map equation compresses bipartite networks better and finds modules that, presumably, capture the network structure better
- The detected communities are shifted more towards a bipartite connection pattern

arXiv:soon

Thank you for your attention!

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